JEE Main Exam 2018 (Paper & Solution)

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PART- A (MATHEMATICS)

Q.1 If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is :

(1) $\frac{9}{2}$ (2) 6 (3) $\frac{7}{2}$ (4) 4

Ans. [1]

Sol. Curve $y^2 = 6x, 9x^2 + by^2 = 16$ intersect orthogonally

Let point of intersection is $\left(\frac{3}{2}t^2, 3t\right)$.

Now, $2y\, m_1 = 6$ & $18x + 2bym_2 = 0$

$\Rightarrow m_1 = \frac{3}{y}$ & $m_2 = \frac{-9x}{by}$

$\Rightarrow m_1 = \frac{3}{3t}$ & $m_2 = \frac{-9\left(\frac{3}{2}t^2\right)}{b(3t)}$

$\Rightarrow m_1 = \frac{1}{t}$ & $m_2 = \frac{-9t}{2b}$

intersect orthogonally Hence, $m_1 \cdot m_2 = -1$

$\Rightarrow \left(\frac{1}{t}\right)\left(\frac{-9t}{2b}\right) = -1$

$\Rightarrow b = \frac{9}{2}$

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Q.2 Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to :

(1) 84 (2) 336 (3) 315 (4) 256

Ans. **[2]**

Sol. **Sol.** $\vec{u} = \lambda \vec{a} + \mu \vec{b}$

$\vec{u} . \vec{a} = 0$, $\vec{u} . \vec{b} = 24$

$\vec{u} = \lambda(2\hat{i} + 3\hat{j} - \hat{k}) + \mu(\hat{j} + \hat{k})$

$\vec{u} = 2\lambda\hat{i} + (3\lambda + \mu)\hat{j} + (-\lambda + \mu)\hat{k}$

$\vec{u} . \vec{a} = 0 \Rightarrow 4\lambda + 9\lambda + 3\mu + \lambda - \mu = 0$

$\Rightarrow$ $7\lambda + \mu = 0$.........(i)

$\vec{u} . \vec{b} \Rightarrow 0 + (3\lambda + \mu) - \lambda + \mu = 24$

$\Rightarrow$ $\lambda + \mu = 12$ ....(ii)

From (i) & (ii)

$\lambda = -2$, $\mu = 14$

$\vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$|\vec{u}|^2 = 336$

Q.3 For each $t \in \mathbf{R}$, let [t] be the greatest integer less than or equal to t. Then

$$\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$$

(1) Does not exist (in **R**)

(2) is equal to 0

(3) is equal to 15

(4) is equal to 120

Ans. **[4]**

Sol. $$\lim_{x \to 0^+} \frac{x(1+2+3+4+5+\ldots+15)}{x} - x\left\{\frac{1}{x}\right\} - x\left\{\frac{2}{x}\right\} - \ldots - 15\text{times}$$

$$= 120 - \lim_{x \to 0^+} x \sum_{r=1}^{15} \left\{ \frac{r}{x} \right\}$$

= 120 – 0 × (a number between 0 to 1) = 120

Q.4 If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 , is :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{4\sqrt{2}}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{2\sqrt{2}}$

Ans. **[3]**

Sol. Equation of two planes is

$2x - 2y + 3z - 2 = 0$

$x - y + z + 1 = 0$

Let D.R. of intersection line be a_1, b_1, c_1.

$\therefore \frac{a_1}{1} = \frac{b_1}{1} = \frac{c_1}{0}$

If line of intersection intersect yz plane $\therefore$ $x = 0 \Rightarrow y = 5, z = 4$

One of the points on the line is (0, 5, 4)

Again equation of two planes are

$x + 2y - z - 3 = 0$

$3x - y + 2z - 1 = 0$

Again D.R. of intersection line a_2, b_2, c_2.

$\therefore \frac{a_2}{3} = \frac{b_2}{-5} = \frac{c_2}{-7}$

Let D.R. of plane passage through both intersecting line is a,b,c

$\therefore \frac{a}{-7} = \frac{b}{7} = \frac{c}{-8}$

$\therefore$ Equation of plane $-7x + 7y - 8z - 3 = 0$

$\therefore$ Perpendicular distance from origin = $\frac{1}{3\sqrt{2}}$

Q.5 The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{8}$ (3) $\frac{\pi}{2}$ (4) 4π

Ans. **[1]**

Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x} dx \ldots$ ... (i)

$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^{-x}} dx = \int_{-\pi/2}^{\pi/2} \frac{2^x \sin^2 x}{1+2^x} dx \ldots$ ... (ii)

Adding eq. (i) and (ii) :

$$2I = \int_{-\pi/2}^{\pi/2} \frac{(1+2^x) \cdot \sin^2 x}{1+2^x} dx$$

$$\Rightarrow 2I = \int_{-\pi/2}^{\pi/2} \left(\frac{1-\cos 2x}{2}\right) dx$$

$$\Rightarrow I = \frac{1}{4}\left[x - \frac{\sin 2x}{2}\right]_{-\pi/2}^{\pi/2}$$

$$\Rightarrow I = \frac{1}{4}\left[\left(\frac{\pi}{2} - 0\right) - \left(-\frac{\pi}{2} - 0\right)\right]$$

$$\Rightarrow \boxed{I = \frac{\pi}{4}}$$

Q.6 Let g (x) = cos x^2, $f(x) = \sqrt{x}$, and $\alpha, \beta\ (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is :

(1) $\frac{1}{2}(\sqrt{2}-1)$ (2) $\frac{1}{2}(\sqrt{3}-1)$ (3) $\frac{1}{2}(\sqrt{3}+1)$ (4) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

Ans. **[2]**

Sol. $gf(x) = \cos x$

$\alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

$$\int_{\pi/6}^{\pi/3} \cos x dx = [\sin x]_{\pi/6}^{\pi/3}$$

$$= \frac{\sqrt{3}-1}{2}$$

Q.7 If sum of all the solutions of the equation $8\cos x \cdot \left(\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in [0, π] is kπ, then k is equal to :

(1) $\frac{20}{9}$ (2) $\frac{2}{3}$ (3) $\frac{13}{9}$ (4) $\frac{8}{9}$

Ans. **[3]**

Sol. $\cos x\left[\cos\left(\frac{\pi}{6}+x\right)\cdot\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right]=1$

$\Rightarrow 8\cos x\left[\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right]=1$

$\Rightarrow \cos 3x = \frac{1}{2}$

$\therefore x \in [0,\pi] \Rightarrow 3x \in [0,3\pi]$

$\therefore 3x = \frac{\pi}{3}, 2\pi-\frac{\pi}{3}, 2\pi+\frac{\pi}{3}$

$\boxed{x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}}$

Sum of all solution $= \frac{13\pi}{9}$

Q.8 Let $f(x)=x^2+\frac{1}{x^2}$ and $g(x)=x-\frac{1}{x}, x\in \mathbf{R}-\{-1,0,1\}$. If $h(x)=\frac{f(x)}{g(x)}$, then the local minimum value of h(x) is :

(1) $2\sqrt{2}$ (2) 3 (3) – 3 (4) $-2\sqrt{2}$

Ans. **[1]**

Sol. $f(x)=x^2+\frac{1}{x^2}; g(x)=x-\frac{1}{x}$

$$\Rightarrow h(x)=\frac{f(x)}{g(x)}=\frac{x^2+\frac{1}{x^2}}{x-\frac{1}{x}}$$

$$\Rightarrow h(x)=\left(x-\frac{1}{x}\right)+\frac{2}{\left(x-\frac{1}{x}\right)}$$

Let $x-\frac{1}{x}=\alpha$

$$\Rightarrow \quad h(\alpha)=\alpha+\frac{2}{\alpha}=\left(\sqrt{\alpha}-\sqrt{\frac{2}{\alpha}}\right)^2+2\sqrt{2}$$

$$\Rightarrow h(\alpha)\geq 2\sqrt{2}$$

Q.9 The integral

$$\int \frac{\sin^2 x\cos^2 x}{\left(\sin^5 x+\cos^3 x\sin^2 x+\sin^3 x\cos^2 x+\cos^5 x\right)^2}dx \quad \text{is equal to :}$$

(1) $\frac{-1}{1+\cot^3 x}+C$

(2) $\frac{1}{3\left(1+\tan^3 x\right)}+C$

(3) $\frac{-1}{3\left(1+\tan^3 x\right)}+C$

(4) $\frac{1}{1+\cot^3 x}+C$

(where C is a constant of integration)

Ans. **[3]**

Sol.
$$=\int \frac{\sin^2 x\cdot\cos^2 x}{\left(\sin^2 x\left(\sin^3 x+\cos^3 x\right)+\cos^2 x\left(\sin^3 x+\cos^3 x\right)\right)^2}dx$$

$$=\int \frac{\sin^2 x\cdot\cos^2 x}{\left(\sin^3 x+\cos^3 x\right)^2\left(\sin^2 x+\cos^2 x\right)^2}dx$$

$$= \int \frac{\sin^2 x \cdot \cos^2 x}{\left(\sin^3 x + \cos^3 x\right)^2} dx$$

$$= \int \frac{\tan^2 x \cdot \sec^2 x}{\left(\tan^3 x + 1\right)^2} dx$$

Let $1 + \tan^3 x = t$

$3\tan^2 x \cdot \sec^2 x dx = dt$

$$\Rightarrow \frac{1}{3}\int \frac{dt}{t^2}$$

$$\Rightarrow \frac{1}{3}\left(-\frac{1}{t}\right) + C$$

$$\Rightarrow -\frac{1}{3\left(1 + \tan^3 x\right)} + C$$

Q.10 A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is :

(1) $\frac{3}{4}$ (2) $\frac{3}{10}$ (3) $\frac{2}{5}$ (4) $\frac{1}{5}$

Ans. **[3]**

Sol. Given 4 red ball and 6 black ball.

Case – I

If first drawn ball is red, then :

$\frac{4}{10} \times \frac{6}{12} = \frac{24}{120}$ ($\because$ 2 Red balls are added)

So, in box (6 R, 6 B)

Case – II

If first drawn is black, then :

$\frac{6}{10} \times \frac{4}{12} = \frac{24}{120}$ ($\because$ 2 Black balls are added)

So, in box (4 R, 8 B)

$\because \quad P(R) = \frac{24}{120} + \frac{24}{120} = \frac{48}{120} = \frac{2}{5}$

Q.11 Let the orthocenter and centroid of a triangle be $A(-3,5)$ and $B(3,3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is :

(1) $\frac{3\sqrt{5}}{2}$ (2) $\sqrt{10}$ (3) $2\sqrt{10}$ (4) $3\sqrt{\frac{5}{2}}$

Ans. **[4]**

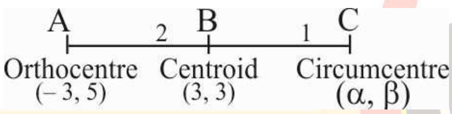
Sol. Given :

Orthocentre = A = (– 3, 5)

Centroid = B = (3, 3)

and C is circumcentre.

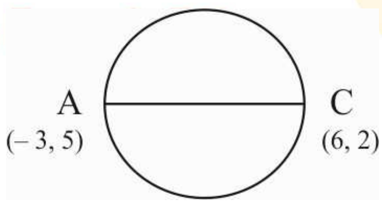
We know that



$\Rightarrow \frac{2\alpha - 3}{3} = 3$ and $\frac{2\beta + 5}{3} = 3$

$\Rightarrow \alpha = 6$ and $\beta = 2$

So, C (6, 2)



Distance AC $= \sqrt{(6+3)^2 + (2-5)^2}$

$= \sqrt{81 + 9}$

$= \sqrt{90}$

Radius $= \sqrt{\frac{90}{4}} = 3\sqrt{\frac{5}{2}}$

Q.12 If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is :

(1) 95 (2) 195 (3) 185 (4) 85

Ans. **[1]**

Sol. T = 0

$\Rightarrow$ y – 2x – 5 = 0, It is also tangent to circle having centre (– 8, – 6) and $r = \sqrt{100 - c}$

Applying condition of tangency c = 95.

Q.13 If $\alpha, \beta \in \mathbf{C}$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to :

(1) 2 (2) – 1 (3) 0 (4) 1

Ans. **[4]**

Sol. $\because \ \alpha + \beta = 1$

$\alpha \cdot \beta = 1$

$\Rightarrow \ \alpha = -\left(\frac{-1-\sqrt{3}i}{2}\right)$

$\Rightarrow \ \alpha = -\omega^2$

$\Rightarrow \ \beta = \frac{1-\sqrt{3}i}{2} \Rightarrow \beta = -\left(\frac{1-\sqrt{3}i}{2}\right)$

$\Rightarrow \ \beta = -\omega$

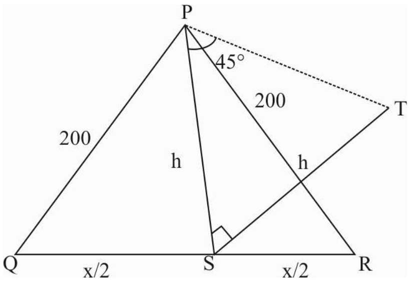
$\Rightarrow \alpha^{101} + \beta^{107} = 1$

Q.14 PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is :

(1) $50\sqrt{2}$ (2) 100 (3) 50 (4) $100\sqrt{3}$

Ans. **[2]**

Sol. $\tan 30° = \frac{ST}{QS} \Rightarrow h = \frac{x}{2\sqrt{3}}$



Also, $PQ^2 = PS^2 + QS^2$

$$40000 = h^2 + \frac{x^2}{4} \Rightarrow 40000 = \frac{x^2}{12} + \frac{x^2}{4} \Rightarrow x = 100\sqrt{12}$$

& h = 100

Q.15 If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, \ldots\ldots, x_9$ is :

(1) 3 (2) 9 (3) 4 (4) 2

Ans. **[4]**

Sol. $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$

$\because$ Standard Deviation (S.D.) $=\sqrt{\text{Var.}(x)}$

Hence, Var. (x) $=\frac{1}{9}\sum_{i=1}^{9}(x_i-5)^2-\left(\sum_{i=1}^{9}\frac{x_i-5}{9}\right)^2$

$\Rightarrow \frac{45}{9}-\frac{81}{81}\Rightarrow\frac{405-81}{81}$

$\Rightarrow \frac{324}{81}=4$

$\because$ Now, Var. (x) = 4

Hence, S. D. $=\sqrt{4}=2$

Q.16 The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5, (x>1)$ is :

(1) 2 (2) – 1 (3) 0 (4) 1

Ans. **[1]**

Sol. $2\left[{}^5C_0x^5+{}^5C_2x^3\left(x^3-1\right)+{}^5C_4x\left(x^3-1\right)^2\right]$

Hence sum of coefficient of odd terms

$2\left[{}^5C_0-{}^5C_2+{}^5C_4+{}^5C_4\right]=2$

Q.17 Tangents are drawn to the hyperbola $4x^2-y^2=36$ at the point P and Q. If these tangents intersect at the point T (0, 3) then the area (in sq. units) of Δ PTQ is :

(1) $36\sqrt{5}$ (2) $45\sqrt{5}$ (3) $54\sqrt{3}$ (4) $60\sqrt{3}$

Ans. **[2]**

Sol. $\frac{x^2}{9}-\frac{y^2}{36}=1$

Tangent of equation of slope (m) is $y=mx+\sqrt{a^2m^2-b^2}$

It passes through (0, 3) Hence m = $\pm\sqrt{5}$

Tangent touches hyperbola at point $\left(-\frac{ma^2}{c},\frac{-b^2}{c}\right)$: $c=\sqrt{a^2m^2-b^2}=3$

$\therefore P=\left(-3\sqrt{5},-12\right)$ & $Q=\left(3\sqrt{5},-12\right)$

$\therefore$ Area of triangle of TPQ = $45\sqrt{5}$

Q.18 From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is :

(1) at least 750 but less than 1000

(2) at least 1000

(3) less than 500

(4) at least 500 but less than 750

Ans. **[2]**

Sol. $={}^6C_4\cdot{}^3C_1\cdot 4!=1080$

Q.19 If the system of linear equations:

$x+ky+3z=0$

$3x+ky-2z=0$

$2x+4y-3z=0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

(1) 30 (2) – 10 (3) 10 (4) – 30

Ans. **[3]**

Sol. $\begin{vmatrix}1 & k & 3\\ 3 & k & -2\\ 2 & 4 & -3\end{vmatrix}=0\Rightarrow k=11$

Now solving the given equations :

$\frac{xz}{y^2}=10$

Q.20 If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to :

(1) (4, 5) (2) (– 4, – 5) (3) (– 4, 3) (4) (– 4, 5)

Ans. **[4]**

Sol. By putting x = 0

$$\begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A = -4$$

By putting x = 1

$$\begin{vmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{vmatrix} = (A+B)(1-A)^2 \Rightarrow B = 5$$

Q.21 Two sets A and B are as under :

$A = \{(a,b) \in R \times R : |a-5| < 1 \text{ and } |b-5| < 1\};$

$B = \{(a,b) \in R \times R : 4(a-6)^2 + 9(b-5)^2 \le 36\}$

Then :

(1) neither $A \subset B$ nor $B \subset A$ (2) $B \subset A$

(3) $A \subset B$ (4) $A \cap B = \phi$ (an empty set)

Ans. **[3]**

Sol. For set A,

$-1 < a-5 < 1 \Rightarrow a \in (4,6)$

$-1 < b-5 < 1 \Rightarrow b \in (4,6)$

Set B represents an ellipse covering larger subset and also all the value of set A satisfies set B.

$\therefore A \subset B$.

Q.22 Tangents and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P,A and B and $\angle CPB = \theta$ then a value of $\tan\theta$ is :

(1) $\frac{4}{3}$ (2) $\frac{1}{2}$ (3) 2 (4) 3

Ans. **[3]**

Sol. Parabola $y^2 = 16x$ & given point P(16, 16)

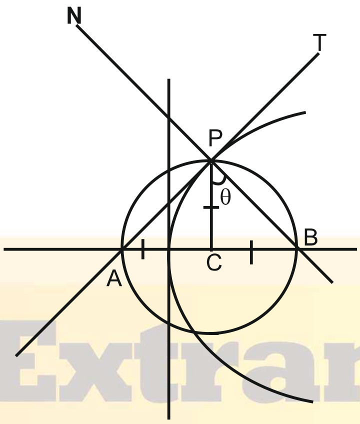
Parametric point $\left(4t^2, 8t\right) \Rightarrow t = 2$

Equation of tangent $T : x - 2y + 16 = 0$

Equation of normal $N : 2x + y - 48 = 0$

Axis of parabola is $y = 0$

So $A(-16, 0)$ & $B(24, 0)$



$m_{PC} = \frac{4}{3}, m_{PB} = -2$

$\tan\theta = 2$

Q.23 Let $S = \left\{t \in R : f(x) = |x - \pi| . \left(e^{|x|} - 1\right) \sin|x| \text{ is not differentiable at } t\right\}$. Then the set S is equal to:

(1) $\{0, \pi\}$ (2) Φ (an empty set) (3) $\{0\}$ (4) $\{\pi\}$

Ans. **[2]**

Sol. As per rule of multiplicity of roots, we can see the function has more than one root at x = 0 and π. Therefore, the function is derivable at all the points.

Q.24 The Boolean expression

$\sim(p \vee q) \vee (\sim P \vee q)$ is equivalent to :

(1) ~ q (2) ~ p (3) p (4) q

Ans. **[2]**

Sol. $\sim(p \vee q) \vee (\sim p \wedge q)$

$= (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$= \sim p \wedge (\sim q \vee q)$

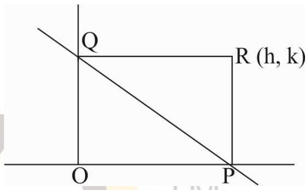
$= \sim p$

Q.25 A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is :

(1) $3x + 2y = 6xy$ (2) $3x + 2y = 6$ (3) $2x + 3y = xy$ (4) $3x + 2y = xy$

Ans. **[4]**

Sol. $y - 3 = m(x - 2)$



So, $P\left(2 - \frac{3}{m}, 0\right)$ & $Q(0, 3 - 2m)$

Let R (h, k)

$\Rightarrow h = 2 - \frac{3}{m} \Rightarrow \frac{1}{m} = \frac{2-h}{3}$ … (i)

$k = 3 - 2m \Rightarrow m = \frac{3-k}{2}$ …(ii)

Eq. (i) + (ii)

$\Rightarrow 1 = \left(\frac{2-h}{3}\right)\left(\frac{3-k}{2}\right)$

$\Rightarrow 6 = (2 - x)(3 - y)$

$\Rightarrow \boxed{3x + 2y = xy}$

Q.26 Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ if B – 2A = 100λ, then λ is equal to:

(1) 496 (2) 232 (3) 248 (4) 464

Ans. **[3]**

Sol. $A = (1^2 + 2^2 + ... + 20^2) + 2^2(1^2 + 2^2 + ... + 10^2) = 4410$

$B = (1^2 + 2^2 + ... + 40^2) + 2^2(1^2 + 2^2 + ... + 20^2) = 33620$

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Now

$B-2A=100\lambda$

Hence $\lambda=248$

Q.27 Let y = y(x) be the solution of the differential equation

$\sin x\dfrac{dy}{dx}+y\cos x=4x, x\in(0,\pi)$, If $y\left(\dfrac{\pi}{2}\right)=0$ then $y\left(\dfrac{\pi}{6}\right)$ is equal to :

(1) $-\dfrac{4}{9}\pi^2$ (2) $\dfrac{4}{9\sqrt{3}}\pi^2$ (3) $\dfrac{-8}{9\sqrt{3}}\pi^2$ (4) $-\dfrac{8}{9}\pi^2$

Ans. **[4]**

Sol. $\sin x\dfrac{dy}{dx}+y\cos x=4x, x\in(0,\pi)$

$\dfrac{dy}{dx}+y\cot x=4x\operatorname{cosec}x$

$I.F.=e^{\int\cot x.dx}$

$y\sin x=\int\sin x(4x\operatorname{cosec}x)dx$

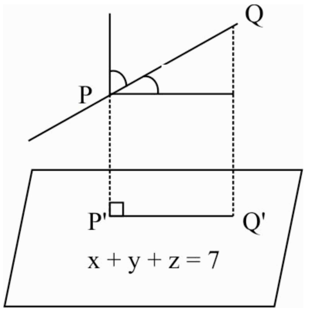
$y\sin x=2x^2+c \Rightarrow 0=2\times\dfrac{\pi}{4}+c\Rightarrow c=\dfrac{-\pi^2}{2}$

$y\sin x=2x^2-\dfrac{\pi}{2} \Rightarrow y\left(\dfrac{\pi}{6}\right)=\dfrac{-8\pi^2}{9}$

Q.28 The length of the projection of the line segments joining the point (5, –1, 4) and (4,–1, 3)on the plane, x + y + z = 7 is :

(1) $\sqrt{\dfrac{2}{3}}$ (2) $\dfrac{2}{\sqrt{3}}$ (3) $\dfrac{2}{3}$ (4) $\dfrac{1}{3}$

Ans. **[1]**



Sol. P (5, – 1, 4), Q (4, – 1, 3)

$PQ=\sqrt{(5-4)^2+(-1-(-1))^2+(4-3)^2}=\sqrt{2}$

& D.R. of PQ = 1, 0, 1

If angle between Plane and line PQ is θ.

$\therefore \quad \sin\theta = \dfrac{1+0+1}{\sqrt{2}\sqrt{3}} = \sqrt{\dfrac{2}{3}}$

$\therefore \quad \cos\theta = \sqrt{1-\dfrac{2}{3}} = \sqrt{\dfrac{1}{3}}$

$P'Q' = PQ\cos\theta$

$= \sqrt{\dfrac{2}{3}}$

Q.29 Let $S = \left\{x \in R : x \geq 0 \text{ and } 2|\sqrt{x}-3| + \sqrt{x}\left(\sqrt{x}-6\right)+6=0\right\}$ Then S :

(1) contains exactly four elements

(2) is an empty set

(3) contains exactly one element

(4) contains exactly two elements

Ans. **[4]**

Sol. $2|\sqrt{x}-3| + \sqrt{x}\left(\sqrt{x}-6\right)+6=0;\ x \geq 0$

Case – I

Let $\sqrt{x} \geq 3$

$2\sqrt{x}-6+x-6\sqrt{x}+6=0$

$\therefore \quad x-4\sqrt{x}=0$

$\sqrt{x}\left(\sqrt{x}-4\right)=0$

$\therefore \quad \sqrt{x}=0$ or $\sqrt{x}=4$

$\times$ $\quad$ $\boxed{x=16}$

Case – II

Let $\sqrt{x} < 3$

$-2\sqrt{x}+6+x-6\sqrt{x}+6=0$

$x-8\sqrt{x}+12=0$

$\therefore \quad \left(\sqrt{x}-2\right)\left(\sqrt{x}-6\right)=0$

$\therefore \quad \sqrt{x}=2 \qquad \sqrt{x}=6$

$\boxed{x=4} \qquad \times$

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Contain exactly two elements.

Q.30 Let $a_1, a_2, a_3......, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$ if $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to.

(1) 33 (2) 66 (3) 68 (4) 34

Ans. **[4]**

Sol. Sum = $13a_1 + (2\times12\times13)d = 416 \Rightarrow a_1 + 26d = 32$ … (i)

$a_9 + a_{43} = 66 \Rightarrow a_1 + 25d = 33$ … (ii)

On solving, d = 1, a = 8

$a_1^2 + a_2^2 + \ldots + a_{17}^2 = 8^2 + 9^2 + \ldots + 24^2 = (1^2 + 2^2 + \ldots + 24^2) - (1^2 + 2^2 + \ldots + 7^2)$

$\Rightarrow$ 4760 = 34 × 140

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